

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

4037/11

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

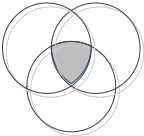
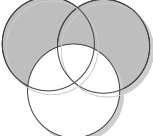
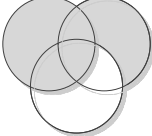
Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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<p>1</p> $\text{LHS} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}$ $= \frac{\sin \theta(1 + \sin \theta) + \cos^2 \theta}{\cos \theta(1 + \sin \theta)}$ $= \frac{1 + \sin \theta}{\cos \theta(1 + \sin \theta)}$ $= \frac{1}{\cos \theta} \text{ leading to } \sec \theta$ <p>Alternative solution:</p> $\text{LHS} = \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta(1 - \sin \theta)}{(1 + \sin \theta)(1 - \sin \theta)}$ $= \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta(1 - \sin \theta)}{\cos^2 \theta}$ $= \frac{\sin \theta}{\cos \theta} + \frac{(1 - \sin \theta)}{\cos \theta}$ $= \frac{1}{\cos \theta} \text{ leading to } \sec \theta$ <p>Alternative solution:</p> $\text{LHS} = \frac{\tan \theta(1 + \sin \theta) + \cos \theta}{1 + \sin \theta}$ $= \frac{\frac{\sin \theta}{\cos \theta} + \frac{\sin^2}{\cos \theta} + \cos \theta}{1 + \sin \theta}$ $= \frac{\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta(1 + \sin \theta)}$ $= \frac{1 + \sin \theta}{\cos \theta(1 + \sin \theta)}$ $= \frac{1}{\cos \theta} \text{ leading to } \sec \theta$	<p>B1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>DM1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>DM1</p> <p>A1</p>	<p>B1 for use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$</p> <p>M1 for attempt to obtain a single fraction</p> <p>DM1 for use of $\sin^2 \theta + \cos^2 \theta = 1$</p> <p>A1 for ‘finishing off’</p> <p>B1 for use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$</p> <p>M1 for multiplication by $(1 - \sin \theta)$</p> <p>DM1 for use of $\sin^2 \theta + \cos^2 \theta = 1$</p> <p>A1 for ‘finishing off’</p> <p>M1 for attempt to obtain a single fraction</p> <p>B1 for use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$</p> <p>DM1 for use of $\sin^2 \theta + \cos^2 \theta = 1$</p> <p>A1 for ‘finishing off’</p>
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<p>2 (i)</p> <p>(ii)</p>	$ a = \sqrt{4^2 + 3^2} = 5$ $ b + c = \sqrt{(-3)^2 + 4^2} = 5$ $\lambda \begin{pmatrix} 4 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 7 \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ $4\lambda + 2\mu = -35 \text{ and } 3\lambda + 2\mu = 14$ leading to $\lambda = -49, \mu = 80.5$	<p>M1</p> <p>A1</p> <p>M1</p> <p>DM1</p> <p>A1</p>	<p>M1 for finding the modulus of either a or b + c</p> <p>A1 for completion</p> <p>M1 for equating like vectors and obtaining 2 linear equations</p> <p>DM1 for solution of simultaneous equations</p> <p>A1 for both</p>
<p>3 (a)</p> <p>(b) (i)</p> <p>(ii)</p>	<p>(i) </p> <p>(ii) </p> <p>(iii) </p> <p>2</p> <p>0</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1</p>	<p>B1 for each</p>
<p>4</p>	$k(4x - 3) = 4x^2 + 8x - 8$ $4x^2 + x(8 - 4k) + 3k - 8 = 0$ $b^2 - 4ac = (8 - 4k)^2 - 16(3k - 8)$ $= 16k^2 - 112k + 192$ $b^2 - 4ac < 0, k^2 - 7k + 12 < 0$ critical values $k = 3, 4$ $\therefore 3 < k < 4$	<p>M1</p> <p>DM1</p> <p>DM1</p> <p>A1</p> <p>A1</p>	<p>M1 for equating the line and the curve and attempt to obtain a quadratic equation in k</p> <p>DM1 for use of $b^2 - 4ac$ with k</p> <p>DM1 for solution of a 3 term quadratic equation, dependent on both previous M marks</p> <p>A1 for both critical values</p> <p>A1 for the range</p>
<p>5 (i)</p> <p>(ii)</p> <p>(iii)</p>	$\frac{dy}{dx} = 2xe^{x^2}$ $\frac{1}{2}e^{x^2}$ $\left(\frac{1}{2}e^4\right) - \left(\frac{1}{2}\right) = 26.8$	<p>B1B1</p> <p>M1A1</p> <p>DM1</p> <p>A1</p>	<p>B1 for e^{x^2}, B1 for $2xe^{x^2}$</p> <p>M1 for ke^{x^2} A1 for $\frac{1}{2}e^{x^2}$</p> <p>DM1 for correct use of limits</p> <p>A1 for 26.8, allow exact value</p>

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<p>6 (i)</p> $\mathbf{AB} = \begin{pmatrix} 10 & 19 \\ 32 & 37 \\ 14 & 14 \end{pmatrix}$ <p>(ii)</p> $\mathbf{B}^{-1} = \frac{1}{7} \begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$ <p>(iii)</p> $2 \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -22 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix} \begin{pmatrix} -1.5 \\ -11 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 3.5 \\ -17.5 \end{pmatrix}$ <p>$x = 0.5, y = -2.5$</p>	<p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>M1 for at least 3 correct elements of a 3×2 matrix</p> <p>A1 for all correct</p> <p>B1 for $\frac{1}{7}$, B1 for $\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$</p> <p>M1 for obtaining in matrix form</p> <p>M1 for pre-multiplying by \mathbf{B}^{-1}</p> <p>A1 for both</p>
<p>7 (i)</p> $y = 2x^2 - \frac{1}{x+1} (+c)$ <p>when $x = \frac{1}{2}, y = \frac{5}{6}$ so $\frac{5}{6} = \frac{1}{2} - \frac{2}{3} + c$</p> <p>leading to $c = 1$</p> $\left(y = 2x^2 - \frac{1}{x+1} + 1 \right)$ <p>(ii)</p> <p>When $x = 1, y = \frac{5}{2}$</p> $\frac{dy}{dx} = \frac{17}{4} \text{ so gradient of normal} = -\frac{4}{17}$ <p>Equation of normal $y - \frac{5}{2} = -\frac{4}{17}(x - 1)$</p> $(8x + 34y - 93 = 0)$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>DM1</p> <p>A1</p>	<p>B1 for each correct term</p> <p>M1 for attempt to find $+c$, must have at least 1 of the previous B marks</p> <p>Allow A1 for $c = 1$</p> <p>M1 for using $x = 1$ in their (i) to find y</p> <p>B1 for gradient of normal</p> <p>DM1 for attempt at normal equation</p> <p>A1 – allow unsimplified (fractions must not contain decimals)</p>

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10 (a)	1 digit even numbers	2	B1		
	2 digit even numbers	$4 \times 2 = 8$	B1		
	3 digit even numbers	$3 \times 3 \times 2 = 18$	B1		
	Total = 28		B1		
	(b) (i)	3M 5W = 35		B1	
		4M 4W = 175		B1	
		5M 3W = 210		B1	
	Total = 420		B1	B1 for addition to obtain final answer, must be evaluated.	
	(ii)	or ${}^{12}C_8 - 6M 2W - 7M 1W$			or: as above, final B1 for subtraction to get final answer
		495 – 70 – 5 = 420			
	Oldest man in, oldest woman out and vice-versa				
	${}^{10}C_7 \times 2 = 240$		B1, B1	B1 for ${}^{10}C_7$, B1 for realising there are 2 identical cases	
	Alternative:				
	1 man out	1 woman in			
	6 men	4 women			
	6M 1W : ${}^6C_6 \times {}^4C_1 = 4$				
	5M 2W : ${}^6C_5 \times {}^4C_2 = 36$				
	4M 3W : ${}^6C_4 \times {}^4C_3 = 60$				
	3M 4W : ${}^6C_3 \times {}^4C_4 = 20$				
	Total = 120		B1	All separate cases correct for B1	
	There are 2 identical cases to consider, so 240 ways in all.		B1	B1 for realising there are 2 identical cases, which have integer values	

Page 7	Mark Scheme	Syllabus	Paper
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<p>11 (a)</p>	$5\sin 2x + 3\cos 2x = 0$ $\tan 2x = -0.6$ $2x = 149^\circ, 329^\circ$ $x = 74.5^\circ, 164.5^\circ$ <p>Alternatives: $\sin(2x + 31^\circ) = 0$ or $\cos(2x - 59^\circ) = 0$</p>	<p>M1 DM1</p> <p>A1,A1</p> <p>M1</p>	<p>In each case the last A mark is for a second correct solution and no extra solutions within the range</p> <p>M1 for use of tan DM1 for dealing with $2x$ correctly</p> <p>A1 for each</p> <p>M1 for either, then mark as above</p>
<p>(b)</p>	$2\cot^2 y + 3\operatorname{cosec} y = 0$ $2(\operatorname{cosec}^2 y - 1) + 3\operatorname{cosec} y = 0$ $2\operatorname{cosec}^2 y + 3\operatorname{cosec} y - 2 = 0$ $(2\operatorname{cosec} y - 1)(\operatorname{cosec} y + 2) = 0$ <p>One valid solution</p> $\operatorname{cosec} y = -2, \sin y = -\frac{1}{2}$ $y = 210^\circ, 330^\circ$ <p>Alternative:</p> $2\frac{\cos^2 y}{\sin^2 y} + \frac{3}{\sin y} = 0$ <p>leads to $2\sin^2 y - 3\sin y - 2 = 0$</p> <p>and $\sin y = -\frac{1}{2}$ only</p> $y = 210^\circ, 330^\circ$	<p>M1</p> <p>M1</p> <p>A1,A1</p> <p>M1</p> <p>M1</p> <p>A1A1</p>	<p>M1 for use of correct identity</p> <p>M1 for attempt to factorise a 3 term quadratic equation</p> <p>A1 for each</p> <p>M1 for use of $\cot y = \frac{\cos y}{\sin y}$ and $\operatorname{cosec} y = \frac{1}{\sin y}$</p> <p>M1 for attempt to factorise a 3 term quadratic equation</p>
<p>(c)</p>	$3\cos(z + 1.2) = 2$ $\cos(z + 1.2) = \frac{2}{3}$ $(z + 1.2) = 0.8411, 5.442, 7.124$ $z = 4.24, 5.92$	<p>M1</p> <p>A1 A1A1</p>	<p>M1 for correct order of operations to end up with 0.8411 radians or better A1 for one of 5.441 or 7.124 (or better) A1 for each valid solution</p>